THE IMPACT OF LIQUID SURGE
WITHIN A TANK

SURGE FORCE MODELING

PREPARED BY

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A theoretical model was developed and calculations have been conducted to estimate the forces caused by a surging liquid load in a tank with no surge control devices. Figure 1 below illustrates the theoretical model as applied to the case of fore-to-aft surge. The details of the model are presented in Appendix 1. Table 1 below shows the results for a tank with a *six foot square foot print*. The tank is modeled with a *four foot mean water depth and a one foot surge amplitude*. The calculations for this test case are shown in Appendix 2.

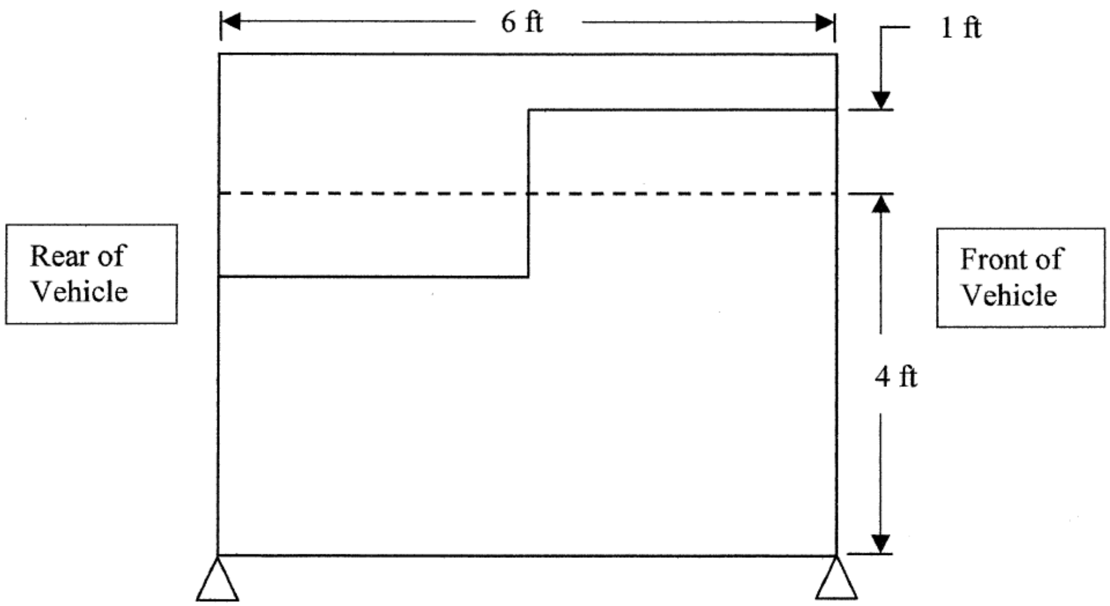


Figure 1. *Schematic diagram of the theoretical model to estimate surge forces. The liquid level is shown at an instant of time where the surge has moved liquid from the rear of the tank to the front of the tank.*

Table 1 reveals that the total weight of the water in the tank is approximately 9000 pounds. This means that under **static conditions** the front tank support and the rear tank support must each support 4500 pounds of water. The tank supports are illustrated in figure 1 by triangle shapes drawn at the base of the tank. When surge occurs, the load carried by the each support will oscillate with time. The primary cause of this oscillation is simply the distribution of the liquid in the tank as it oscillates. For example, in figure 1 it is clear that there is more liquid over the front support than over the rear support. At the instant of time that figure 1 illustrates, the front support will carry 4500 + 1100 = 5600 pounds of water. At the same time, the rear support will carry 4500 - 1100 = 3400 pounds of water. The reaction force of each tank support varies as a sine wave with time as shown in the graph in Figure 3. These estimates do not include the additional effects of vertical liquid acceleration and the thrust of liquid as it move in a u-shaped path from one end of the tank to the other. These could be included in future modeling and might increase the amplitude of the maximum force oscillation amplitude by up to 30%.

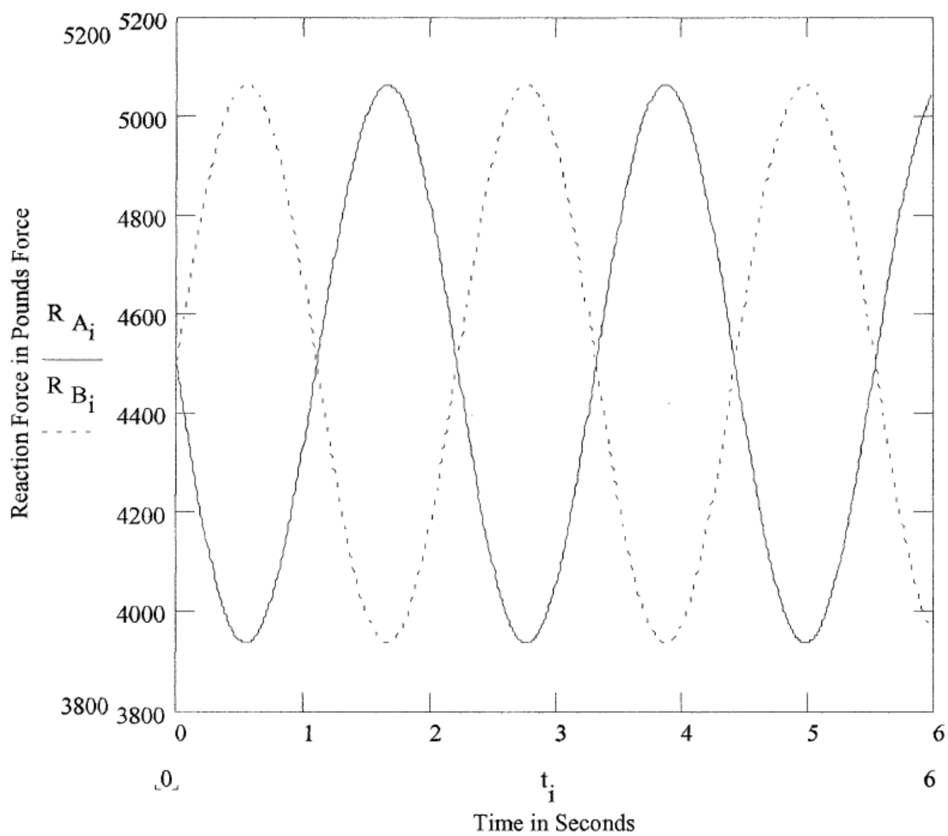


Figure 3. *The vertical reaction force of the tank supports versus time. R_A is the reaction force of the front support and R_B is the reaction force of the rear support.*

Table 1. *Surge modeling results for a tank with no surge control devices.*

Surge Frequency	0.45 (cycles per second)
Surge Time Period	2.2 (seconds)
Total Weight of Water	9000 (pounds force)
Maximum Side-to-Side Surge Force	1700 (pounds force)
Maximum Vertical Force Oscillation Amplitude at Front of Tank	1100 (pounds force)
Maximum Vertical Force Oscillation Amplitude at Rear of Tank	1100 (pound force)

<u>Discussion of Results for a Tank with No Surge Control Devices:</u> The modeling indicates, that for the case shown in Figure 1, the water in the tank will complete one cycle of oscillation in approximately two seconds. The surge force varies as a sine wave with time as shown in the graph in Figure 2. If the water is surging in the fore-to-aft direction then the maximum surge force in that direction will be approximately 1700 pounds force. If the water is surging in the side-to-side direction then the maximum surge force in that direction will be approximately 1700 pounds force. The acceleration and deceleration of the oscillating liquid is the primary cause of the surge force for both the side-to-side and fore-to-aft cases.

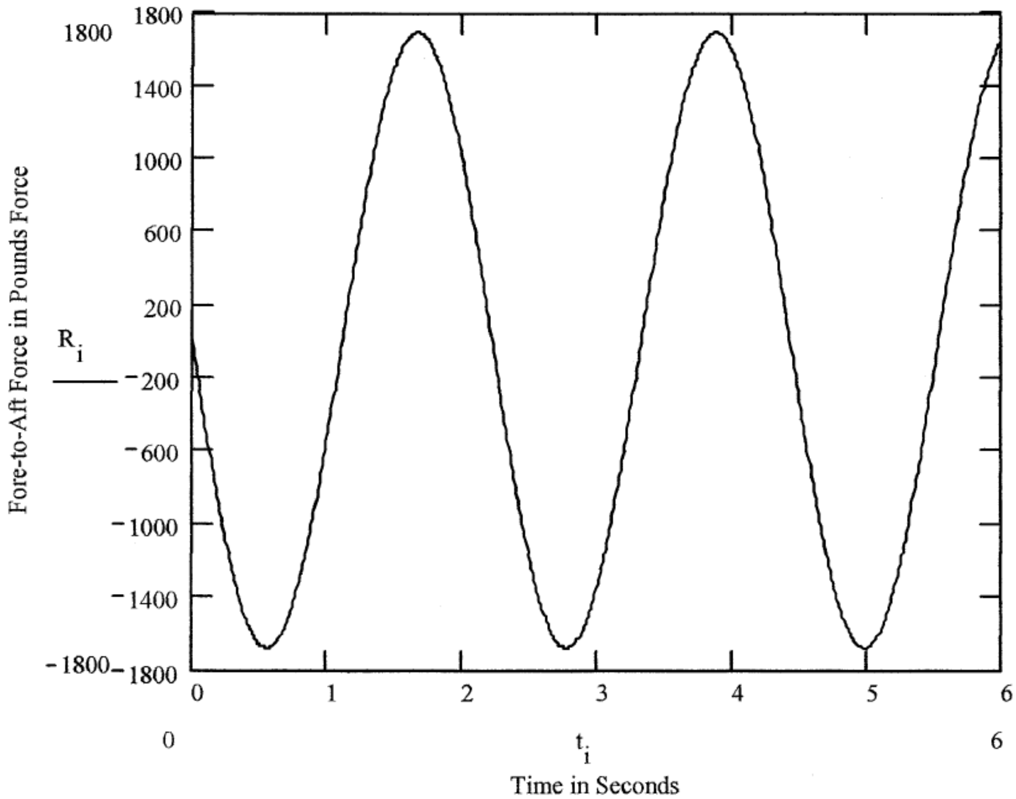


Figure 2. *The fore-to-aft surge force versus time. The side-to-side surge force graph is identical.*

The present model has been developed as a first iteration in a way that will overestimate actual surge forces. For example, figure 1 reveals that the present model treats the oscillating fluid as two vertical columns of fluid that oscillate back and forth. This "U-tube" model is simpler and less time consuming to analyze but will overestimate surge forces.

Discussion of the Action of *Surge Buster*™: The *Surge Buster*™ ellipsoid element will confine regions of liquid oscillation to the approximate size of the ellipsoid. In addition, it is likely that the liquid oscillation from one ellipsoid element to the other will not be in phase. Thus, the force due to acceleration and deceleration of the liquid will approximately cancel when averaged over several elements. The ellipsoid element has an additional benefit in that there are actually two length scales (the major and minor axis of the ellipsoid) that are imposed on the moving fluid. Thus, there will be two time scales of oscillation. This will further reduce the surge force when these effects are averaged over several elements.

Results of Tilt Table Testing: The test was performed by simulating a 1000 gallon tank with 800 gallons of water or 80% of capacity. The test further simulated tank movement consistent with a vehicle speed of 40 miles per hour and an emergency stopping maneuver based on a 60% coefficient of friction.

In the test without any surge control devices:
a Fore-to-Aft Surge Force of 26,480 (pounds force)

In the test with the *Surge Buster* System:
a Fore-to-Aft Surge Force of 860 (pounds force)

This supports the supposition that the liquid oscillation from one ellipsoid element to the other will not be in phase. Thus, the force due to acceleration and deceleration of the liquid will approximately cancel when averaged over several elements.

Calculation and Graphing of Fore-to-Aft Force

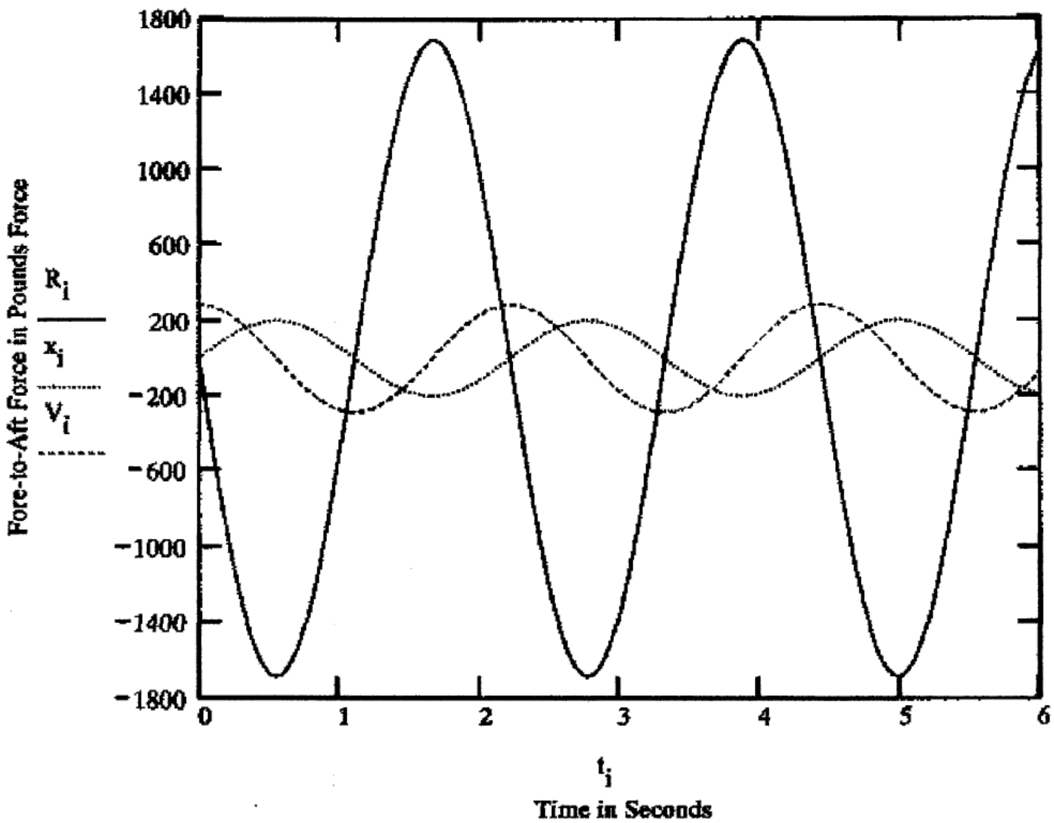
$i := 0..599$ $\qquad t_i := \dfrac{i}{100}$

$C2_i := \sin(w \cdot t_i)$

$R_i := (-p) \cdot s^3 \cdot A \cdot \dfrac{w^2}{2} \cdot C2_i$

$x_i := 200 \cdot A \cdot C2_i$ (motion of right hand water column scaled by factor of 200)

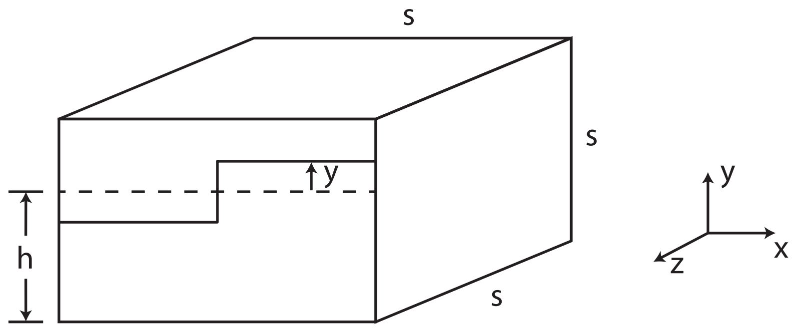
$V_i := 100 \cdot A \cdot w \cdot \cos(w \cdot t_i)$ (velocity of right hand water column scaled by factor of 100)



$R_{max} := \max(R)$ $\qquad R_{max} := 1.687 \cdot 10^3$

APPENDIX 1 - DETAILS OF MODEL

MODEL SCHEMATIC



ESTIMATE FREQUENCY OF OSCILLATION

$m\ddot{y} = F_D$

MASS OF FLUID $m = phs^2$

WEIGHT OF DISPLACED FLUID

$F_D = -p\,g\,(2y)\,\frac{s}{2}\,s$

$F_D = -p\,g\,y\,s^2$

$\cancel{p}h\,\cancel{s^2}\,\ddot{y} = -\cancel{p}\,g\,y\,\cancel{s^2}$

$h\ddot{y} = -gy$

$\ddot{y} + \frac{g}{h}y = 0$

$y = A\sin wt$

$\dot{y} = A\,w\cos wt$

$\ddot{y} = -A\,w^2\sin wt$

$-\cancel{A}\,w^2\,\cancel{\sin wt} + \frac{g}{h}\cancel{A}\,\cancel{\sin wt} = 0$

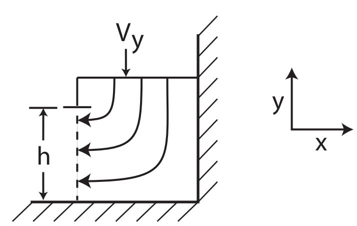
$\Rightarrow w^2 = \frac{g}{h}$, $\boxed{w = \sqrt{\frac{g}{h}}}$

MODEL TO ESTIMATE SIDE-TO-SIDE OR FORE-TO-AFT FORCES

The column of fluid moves down, turns, and then exits in the minus x-direction through an area h x s.

Assume that V_y is uniform along the top.

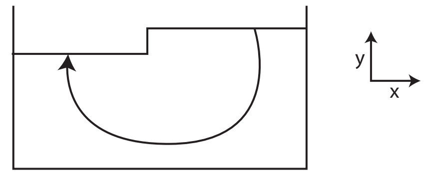
Assume that V_x is constant over a volume h x s x s.



Right Half of Tank

** The thrust of the turning fluid in the x-direction will be zero because of symmetry.

Apply conservation of mass to find V_X.



$$V_y \left(\frac{s}{2}\right)\left(s\right) = V_X\, h\, s$$

$$V_X = \left(\frac{1}{2}\right)\left(\frac{s}{h}\right) V_y$$

Apply conservation of momentum in the x-direction

$$R_X = p \frac{d}{dt} \int_{CV} V_X \, d\forall + \underbrace{\int_{CS} p\, V_X \, V_i \, da}_{0 \sim \text{SEE ABOVE **}}$$

$$R_X = p \frac{d}{dt} \left(V_X \forall\right)$$

$$R_X = p \forall \frac{d}{dt} \left(V_X\right)$$

$$R_X = p\, h\, s^2 \frac{d}{dt} \left(\frac{1}{2} \frac{s}{h} V_y\right)$$

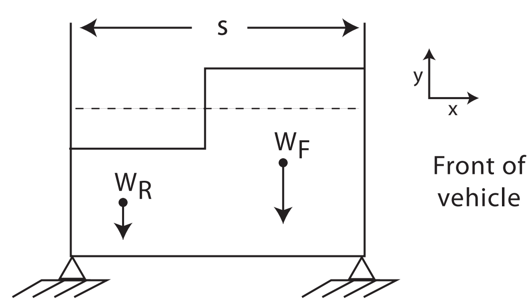
$$R_X = \frac{1}{2} p s^3 \frac{dV_y}{dt} = -\frac{1}{2} p s^3 \, A w^2 \sin wt$$

Units Check

$$\left[R_X\right] = \left(\frac{kg}{m^3}\right)\left(m^3\right)\left(m\right)\left(\frac{1}{s^2}\right) = \frac{kg-m}{s^2} = N$$

Estimate Up-Down force-oscillation on fore and aft tank supports

Apply quasi-steady approximation:
1. Neglect thrust to turn fluid
2. Neglect vertical acceleration of fluid



$$\overset{+}{\curvearrowright} \quad \sum m_A = 0 = \frac{s}{4} W_R + \frac{3s}{4} W_F - R_B s$$

$$\Rightarrow R_B = \frac{1}{4} W_R + \frac{3}{4} W_F$$

$$\sum F_y = 0 = R_A + R_B - W_R - W_F$$

$$R_A = W_R + W_F - R_B$$

$$R_A = \frac{3}{4} W_R + \frac{1}{4} W_F$$

$$W_F = p g \forall_F = p g\, s\left(\frac{s}{2}\right)\left(h + A \sin wt\right)$$

$$W_F = \frac{1}{2} p g s^2 \left(h + A \sin wt\right)$$

$$W_R = \frac{1}{2} p g s^2 \left(h - A \sin wt\right)$$

APPENDIX 2

Surge Calculations For A Tank With No Surge Control - June 2004

$s := 6$ (width of tank and length of tank in feet)

$h := 4$ (height of water in the tank in feet)

$A := 1$ (amplitude of sloshing motion in feet)

$p := 1.94$ (water density ion slugs per foot cubed)

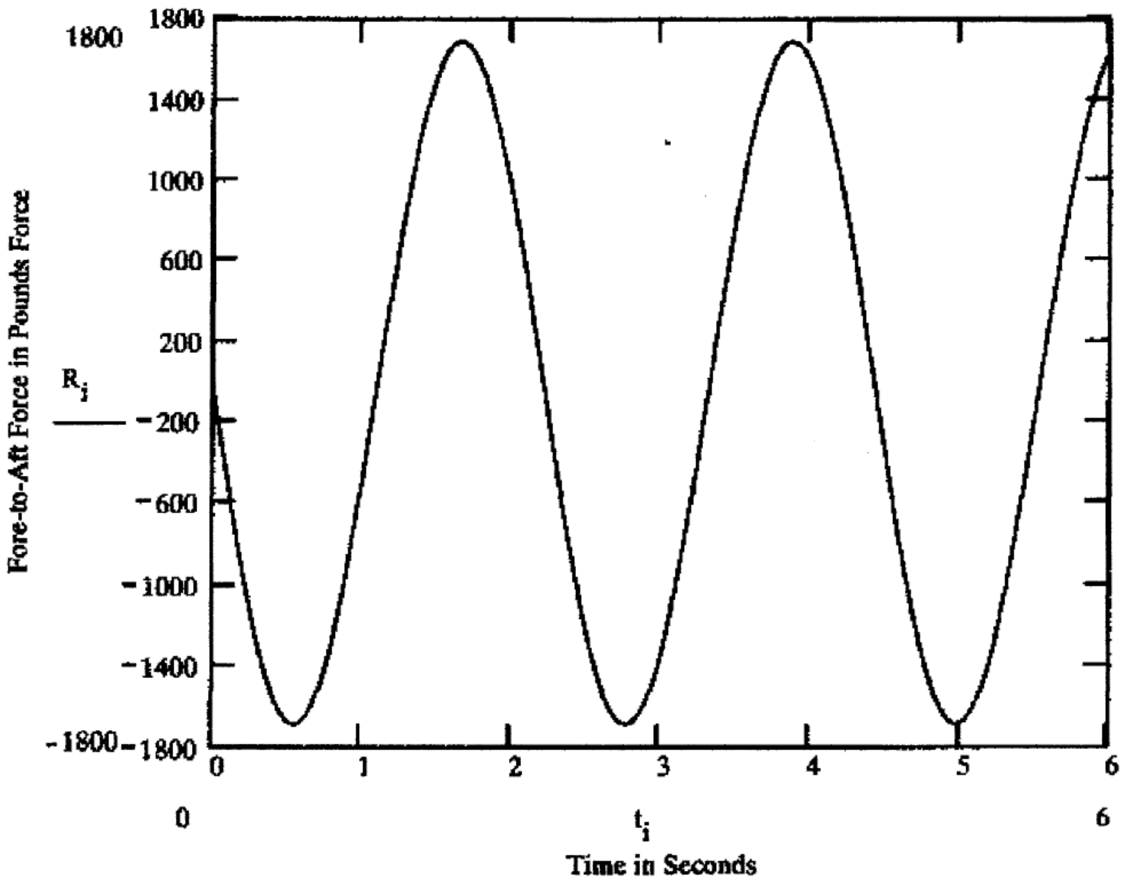
$g := 32.2$ (gravitational acceleration in feet squared per second)

$W := h \cdot s^2 \cdot p \cdot g$ $W = 8.995 \cdot 10^3$ (weight of water in pounds force)

$W1 := \frac{W}{2000}$ $W1 = 4.498$ (weight of water in tons)

Calculation of Surge Oscillation Frequency

$w := \sqrt{\frac{g}{h}}$	$w = 2.837$	(angular frequency of sloshing in radians per second)
$f := \frac{w}{2 \cdot x}$	$f = 0.452$	(frequency of sloshing in Hertz)
$t := \frac{1}{f}$	$t = 2.215$	(period of sloshing in seconds)



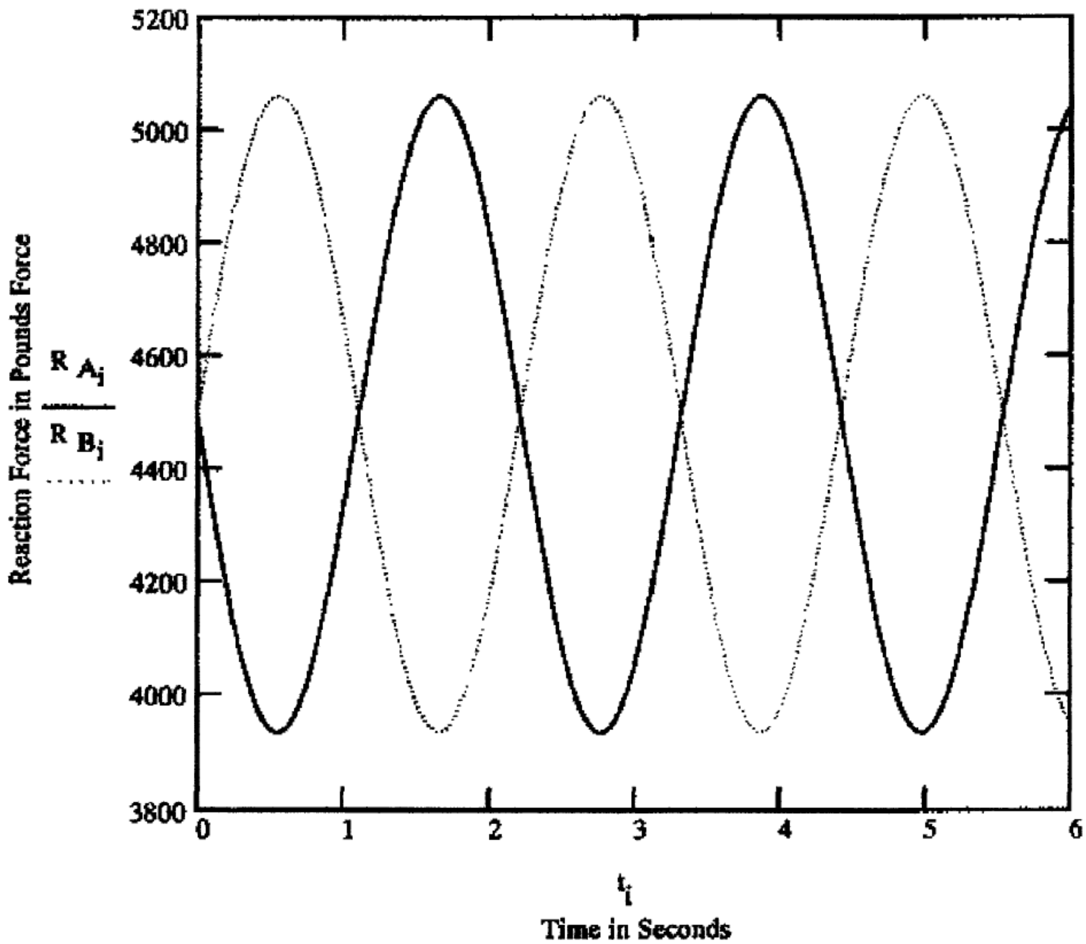
Calculation and Graphing of Vertical Force on Tank Supports

$$W_{f_i} := \frac{p \cdot g \cdot s^2}{2} \cdot (h + A \cdot \sin(w \cdot t_i))$$

$$W_{r_i} := \frac{p \cdot g \cdot s^2}{2} \cdot (h - A \cdot \sin(w \cdot t_i))$$

$$R_{A_i} := 0.75 \cdot W_{r_i} + 0.25 \cdot W_{f_i}$$

$$R_{B_i} := 0.25 \cdot W_{r_i} + 0.75 \cdot W_{f_i}$$



$R_{max} := \max(R_A)$ $R_{max} = 5.06 \cdot 10^3$

$R_{min} := \min(R_A)$ $R_{min} = 3.935 \cdot 10^3$

$R_{pp} := R_{max} - R_{min}$ $R_{pp} = 1.124 \cdot 10^3$

Maximum Thrust to Turn Liquid

$$F_S := p \cdot \frac{s^2}{2} \cdot (w \cdot A)^2$$

$F_S = 281.106$ $\qquad$ $F_S \cdot 2 = 562.212$

Maximum Liquid Acceleration

$a := A \cdot w^2$ $\qquad$ $a = 8.05$

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